"On the Brightness of the Corona of January 22, 1898. Preliminary Note." By H. H. TURNER, D.Sc., F.R.S., Savilian Professor. Received January 18,—Read February 7, 1901.

1. In a former note* I gave some account of measures of brightness made on photographs of the corona of 1893 by Abney's method. The same method has been used on the coronal photographs taken in 1898 and in 1900 (in 1896 none were obtained owing to cloud), and a large number of measures have been made, though the work is not yet complete. Pending the completion and publication of this work, it seems advisable to publish the present note, as one or two results have been arrived at which may be useful to others in the forthcoming eclipse.

2. As regards the method of measurement, sufficient has been said (for the present purpose) in the paper already quoted. It need only be added that in place of the revolving sectors a graduated wedge of gelatine was used to diminish the comparison beam, according to Sir W. Abney's more recent methods. The wedge or sectors are mere intermediaries between the coronal image and the standard squares, and no considerations beyond those of convenience are involved. The wedge is much more convenient, and the work can be done with it twice as rapidly.

3. But a new method has been adopted of representing the results, which, though an elementary change in some respects, has had the important consequence of suggesting a more satisfactory law for the variation of coronal brightness with distance from the sun. The only simple law (so far as I am aware) which has hitherto been formulated was that proposed by Professor Harkness in 1878, viz. :—

$$\text{Brightness} \propto (\text{distance from sun's limb})^{-2}.$$

Visual measures made by Thorpe and Abney in 1886 and 1893 could not be reconciled with this law; though I showed in the paper already quoted that if the distance be measured from a point *within* the limit (about $\frac{1}{5}$ radius within), the law approximately satisfied the photographic measures.

I have now been led to a completely new law, viz. :—

$$\text{Brightness} \propto (\text{distance from sun's } \textit{centre})^{-6},$$

which, though still on trial, is supported by a fair amount of evidence, and the suggestion arose in the following way :—

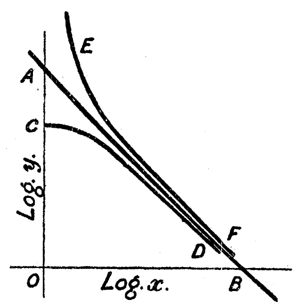
4. The brightness curve in the previous paper was obtained by plotting brightness against distance. This gives a curve of hyperbolic

* 'Roy. Soc. Proc.,' vol. 66, p. 403.

form close to the two axes of reference, and difficult to compare the observations with, for reasons which are tolerably obvious. The curve is still hyperbolic if log (brightness) be plotted against distance; but if the brightness varies as any power of the distance, and we plot log (brightness) against log (distance), we get a straight line, which is particularly easy to compare observations with. The only difficulty is that we must know where to measure our distance from; for if we add or subtract a constant to the distance, it will change the straight line into a curve. And unfortunately the point from which the distance was to be measured seemed just one of the things to be determined.

5. But after some preliminary experiments I found that it was not difficult to find the proper origin from which to measure the distance, by the very condition that the curve was to be a straight line.

FIG. 1.



If in the equation

$$\log y + n \log x = \text{const.}$$

represented by the straight line AB in fig. 1, we write $(x + \alpha)$ for x, then the calculated values of $\log y$, when x is large compared with α, will be nearly the same as before; but when x is small $\log (x + \alpha)$ will be increased, and $\log y$ therefore diminished, and we get a curve such as CD. (If α be negative, we get a curve such as EF.) And a very few trials (perhaps one alone suffices) give the value of α, which will straighten the curve.

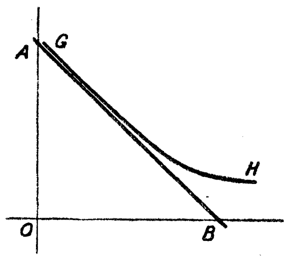
6. These values immediately pointed to the sun's centre as the proper origin for measurement; and when the observations were plotted on this assumption, the curve was practically a straight line, and the slope of this line indicated that the index n was 6, giving the law already stated, viz.:—

$$\text{Brightness} \propto (\text{distance from sun's centre})^{-6}.$$

7. But one further point is to be noted. The curve was practically straight for some distance from the limb, but then always turned

upwards like the curve GH in fig. 2. Now comparing this with CD in fig. 1, it suggests that just as CD could be explained by the addition of a constant to the *distance*, which made a variable alteration in the log distance, so GH may be explained by the addition of a constant to

Fig. 2.



the *brightness*, making a variable alteration in the log brightness. And there is a possible physical cause for this constant addition, viz., the general sky illumination or glare which is added to the coronal brightness. A value of about 0·012 of the average brightness of the full moon for this illumination seems to satisfy requirements for the 1898 photographs.

8. I proceed to give a brief summary of the measures on the photographs of 1898 so far as they have gone.

Four photographs have been selected for measurement, three of them taken by me at Sahdol with exposures of 1 sec., 2 secs. and 20 secs., and one taken by Capt. Hills at Pulgaon with exposure 8 secs. On these, measures have been made along six radii extending approximately N., S., E., W., N.E., and S.W., the last two being as nearly as possible in the direction of the main streamers.

9. The exposures given to the standard squares were all the same. These squares transmit fractions of the light ranging from 0 to 4 on a scale of powers of 2, a range which might be extended with advantage, seeing that measures on the corona can be profitably made over a range of 0 to 7 at least. But the smallness of the range is made up for in practice by the measurement of photographs with different exposures. Thus the longer exposures of 20 secs. and 8 secs. in the above series control the fainter parts of the corona, and the shorter of 1 sec. and 2 secs. control the brighter parts near the limb.

10. In comparing the results from the different plates, it is found that the brightnesses shown by one plate differ from those shown by another in a constant *ratio*. Since the log (brightness) is tabulated this means a constant *difference* between similar numbers for the two plates. Following Sir W. Abney's practice, I have used the base 2 for the logarithms of brightness, and recorded to 0·1, which represents a

ratio of $2^{0 \cdot 1} = 1 \cdot 07$. (The logarithms of distance have been taken to base 10 in the ordinary way.) These differences between the plates may be due to any combination of the following causes:—

(*a.*) Accidental error in exposure to corona. The exposures were made without any mechanism, and the short ones especially may be sensibly in error. Thus the difference between the 1 sec. and 20 secs. exposure is 0·8. If the whole of this be due to accidental error in the 1 sec. exposure, it would mean that the exposure was for 1 sec. $\times\ 2^{-0 \cdot 8} = 0 \cdot 58$ sec. instead of for 1·0 sec., which is not an extravagant supposition.

(*b.*) Accidental error in exposure to squares. This should be much smaller than (*a.*).

(*c.*) Difference in sensitiveness of the film near the edge of the plate where the squares are impressed, and in the centre where the corona is impresssed. There is independent evidence of sensible differences of this kind, and the point is under investigation.

(*d.*) Differences in the behaviour of the candle which impressed the squares on the various plates.

(*e.*) Climatic differences between Sahdol and Pulgaon.

11. It becomes necessary to decide which plate to take as the standard. Cause (*a.*) ought not to affect the 8 secs. and 20 secs. appreciably, but cause (*e.*) may. They differ by 0·5, and we may perhaps take the mean. The corrections to be applied to the plates are then

Plate	I	II	III	IV
Exposure......	1 sec.	2 sec.	8 sec.	20 sec.
Place	Sahdol	Sahdol	Pulgaon	Sahdol
Correction ...	+0·6	−0·2	+0·3	−0·2

If any other selection is preferred, it is easily applicable as a constant to the final numbers.

12. The correction for constant illumination of the plate due to skyglare has been adopted as $2^{-6 \cdot 4}$ moon, taking the moon as equal to 0·02 of a candle at 1 foot. If at any point the corona has a brightness represented by x, meaning $2^x \times$ moon, then the brightness measured on the plate will appear as y where

$$2^x + 2^{-6 \cdot 4} = 2^y.$$

A table was formed giving y in terms of x, of which the following is a portion:—

x.	Correction to x.	y.
−2·0	0·0	−2·0
−3·0	+0·1	−2·9
−4·0	+0·2	−3·8
−5·0	+0·4	−4·6
−6·0	+0·8	−5·2
−7·0	+1·3	−5·7
−8·0	+2·0	−6·0

13. The measures on the plates were then corrected—

(*a.*) For the particular plate, as in § 10;
(*b.*) For the sky-glare, as in § 11;

and compared with the curve

$$\text{brightness} \times (\text{distance})^6 = \text{A}$$

to get the value of the constant A for each of the six radii measured. As above explained, the curve used was a straight line, obtained by plotting log brightness as ordinate and log distance as abscissa. The constants found for the six radii were as follows—adopting as unit of brightness that of the moon (assumed 0·02 candle at 1 foot), and of distance that of the sun's radius, so that the constants represent the brightness of the corona at the sun's limb expressed in moons :—

Radius.	N.	N.E.	E.	S.	S.W.	W.	Mean.
A =	+0·4	+1·9	+1·7	0·0	+2·3	+0·6	+1·15

Thus at the sun's limb the corona is more than twice as bright as the full moon on the average.

14. Finally, the individual measures were compared with the adopted law, with the following results. In the column "Typical Curve" the calculated brightness is given for A = + 0·6, the actual figures for the different streamers differing from this throughout by constants which are easily inferred from the values of A given above.

Table I.—Comparison of Observed Brightness (Photographic) of 1898 Corona with the Law.

Brightness × (distance from Sun's centre)^6 = constant.

(The distances were measured in divisions of 13 to the Sun's radius. The brightnesses are expressed by powers of 2, zero representing Moon's brightness.)

Distance from Sun's centre in radii.	Typical brightness of corona alone.	Typical brightness with "glare" added.	Observed error of formula.						
			Plate.	N.	N.E.	E.	S.	S.W.	W.
1·08	+ 0·1	+0·1	I	+0·6	—	−0·1	+0·7	—	+0·4
1·15	− 0·4	−0·4	I	+0·6	−0·1	0·0	+0·4	−0·7	+0·4
1·23	− 1·0	−1·0	I	+0·2	—	−0·1	+0·2	−0·4	0·0
1·31	− 1·5	−1·5	I	+0·5	+0·1	+0·2	−0·2	−0·3	+0·1
1·38	− 2·0	−2·0	I	−0·4	—	0·0	0·0	0·0	+0·1
1·46	− 2·5	−2·4	I	0·0	+0·1	−0·5	+0·3	−0·1	−0·1
1·61	− 3·3	−3·1	I	−0·3	−0·5	−0·7	—	0·0	—
1·77	− 4·1	−3·8	I	—	−0·1	—	—	+0·2	—
1·92	− 4·9	−4·4	I	—	−0·1	—	—	+0·2	—
1·31	− 1·5	−1·5	II	+0·3	—	—	−0·1	—	—
1·38	− 2·0	−2·0	II	0·0	—	—	−0·3	—	+0·2
1·46	− 2·5	−2·4	II	−0·2	—	—	−0·5	—	−0·1
1·54	− 2·9	−2 8	II	−0·3	+0·5	+0·6	−0·3	—	−0·2
1·61	− 3·3	−3·1	II	−0·4	+0·1	+0·2	−0·3	+0·3	−0·2
1·77	− 4·1	−3·8	II	−0·2	−0·2	+0·1	−0·1	−0·1	−0·1
1·92	− 4·9	−4·4	II	—	−0·2	+0·1	—	+0·1	—
2·15	− 5·8	−5·1	II	+0·1	−0·3	+0·1	—	+0·2	—
2·54	− 7·2	−5·8	II	—	—	—	—	+0·3	—
1·46	− 2·5	−2·4	III	0·0	—	—	+0·1	—	—
1·61	− 3·3	−3·1	III	0·0	—	—	+0·1	—	0·0
1·77	− 4·1	−3·8	III	+0·1	+0·4	+0·2	+0·1	0·0	−0·1
1·92	− 4·9	−4·4	III	−0·1	+0·2	+0·2	+0·1	+0·1	−0·2
2·15	− 5·8	−5·1	III	−0·1	0·0	−0·1	+0·1	0·0	0·0
2·54	− 7·2	−5·8	III	+0·2	+0·2	+0·1	—	0·0	0·0
2·92	− 8·5	−6·1	III	—	+0·4	0·0	—	−0·1	—
1·92	− 4·9	−4·4	IV	+0·1	—	—	+0·3	—	—
2·08	− 5·5	−4·9	IV	+0·2	−0·4	—	+0·4	—	+0·4
2·23	− 6·1	−5·3	IV	+0·2	−0·2	0·0	+0·4	—	+0·2
2·38	− 6·7	−5·6	IV	—	−0.2	+0·1	—	0·0	+0·2
2·54	− 7·2	−5·8	IV	+0·2	−0·2	0·0	+0·5	0·0	+0·2
2·92	− 8·5	−6·1	IV	−0·1	−0·2	+0·1	+0·3	0·0	+0·3
3·31	− 9·6	−6·3	IV	—	−0·1	+0·1	+0·1	−0·1	0·0
3·69	−10·5	−6·3	IV	—	−0·2	0·0	—	−0·3	—
4·08	−11·4	−6·4	IV	—	−0·4	—	—	−0·3	—

15. Considering the irregularity of the coronal structure, we cannot perhaps expect better agreement with any simple law of brightness than is shown by these residuals; and the assumed law, whether it has

any physical significance or not, is, at any rate, a convenient method of expressing the facts. We may now turn to the measures previously given of the 1893 corona,* and see how they accord with this formula. On trial, it is found that a fair accordance can be secured if the constant correction for sky-glare be taken as $2^{-7\cdot8}$ instead of $2^{-6\cdot4}$, and the constants for the four radii measured be

N.	S.	E.	W.	Mean.
−0·1	+0·4	+0·5	+0·1	+0·23

16. With regard to the smaller value for sky-glare, if this depends on the general brightness of the corona itself, we may remark that the 1893 corona was generally fainter, according to the measures, than the 1898 corona, the mean constant for the former being + 0·23, and for the latter + 1·15. The difference is + 0·92, so that the 1898 corona was about twice as bright, and hence twice as bright a sky illumination is not unreasonable.

Table II.—Comparison of Observed Brightness (Photographic) of 1893 Corona with the Law.

Brightness × (distance from Sun's centre)^6 = constant.

(The distances are given in units of the Sun's radius. The brightnesses are expressed by powers of 2; zero representing the Moon's brightness.)

Distance from Sun's centre.	Typical brightness of corona alone.	With "glare" added.	Observed error of formula.			
			N.	S.	E.	W.
1·1	+0·2	+0·2	—	−0·9	—	—
1·2	−0·6	−0·6	−0·1	−0·4	—	—
1·3	−1·2	−1·2	—	−0·1	—	+0·1
1·4	−1·9	−1·9	+0·4	+0·4	−0·3	+0·3
1·5	−2·5	−2·5	+0·2	+0·4	—	+0·5
1·6	−3·0	−2·9	0·0	+0·3	—	—
1·7	−3·6	−3·5	−0·1	+0·4	—	+0·3
1·8	−4·1	−4·0	0·0	+0·2	+0·5	+0·1
1·9	−4·6	−4·4	−0·2	+0·1	—	0·0
2·0	−5·0	−4·8	−0·2	−0·1	—	—
2·1	−5·4	−5·2	−0·2	−0·2	—	−0·3
2·2	−5·8	−5·5	−0·1	−0·2	−0·2	−0·7
2·3	−6·2	−5·8	−0·3	−0·3	—	0·0
2·4	−6·6	−6·1	+0·2	−0·1	—	—
2·5	−7·0	−6·3	+0·1	0·0	—	0·0
2·6	−7·3	−6·5	+0·3	0·0	0·0	+0·1
2·7	−7·6	−6·7	—	+0·1	—	−0·1
2·8	−7·9	−6·8	+0·3	−0·1	—	—
2·9	−8·2	−7·0	—	−0·1	—	+0·1
3·0	−8·5	−7·1	+0·3	−0·1	0·0	+0·1

* 'Roy. Soc. Proc.,' vol. 66, p. 403.

17. The discrepancies are again not large, and some of them may be due to the extrapolation which was necessary for the brighter parts of the corona, the standard squares not having been given a long-enough exposure (as stated in the former paper) to compare with the long exposure of 50 secs. to the corona. Measures on plates with a shorter exposure to the corona will perhaps allow of more accurate results near the sun's limb. Unfortunately no plate is available with an exposure shorter than 5 secs., but measures on this plate, so far as they have gone, indicate a closer accordance with the theoretical formula near the limb. Further measures are, however, required.

18. With the assumed law

$$\text{brightness} = \mathrm{A}r^{-6},$$

where r represents distance from the sun's limb in solar radii, the total brightness of the corona is

$$\int_1^\infty \mathrm{A}r^{-6} \times 2\pi r dr = \tfrac{1}{2}\pi \mathrm{A},$$

the total brightness of the full moon being represented by

$$\int_0^1 2\pi r dr = \pi.$$

Thus the ratio of the total brightness to that of the moon is $\frac{1}{2}$ A. In 1898 the value of A was approximately $2^{1\cdot15} = 2\cdot2$, and thus the whole corona was about equal to the full moon. In 1893 the value of A was $2^{0\cdot23} = 1\cdot2$; and the whole corona was thus about 0·6 of the full moon.

19. But we have omitted the constant illumination of the sky in this integral. If we include a portion of sky extending to distance R from the limb, and B be the value of the constant for "glare," which in 1893 was taken as $2^{-7\cdot8} = 0\cdot0046$, and in 1898 was $2^{-6\cdot4} = 0\cdot012$, then we must add to the above quantities

$$\frac{1}{\pi} \mathrm{B} \int_1^{\mathrm{R}} 2\pi r dr = \mathrm{B}\,(\mathrm{R}^2 - 1) \text{ full moon.}$$

It is not, however, easy to assign a definite value to R.

20. The integral brightness of the corona was measured in 1893 by the late Mr. James Forbes, jun.,* and found to be 1·1 full moon. We find [0·6 + B (R² − 1)] full moon.

If the two quantities be equated, we get

$$\mathrm{B}(\mathrm{R}^2 - 1) = 0\cdot5$$

or

$$\mathrm{R}^2 = 0\cdot5/0\cdot0046$$
$$= 110$$

or

$$\mathrm{R} = 10\cdot5.$$

* 'Phil. Trans.,' A, 1896, p. 433.

Thus, if we suppose that Mr. Forbes measured the total light within a circular area 5° in diameter, which seems a fair supposition,* the two measures of total brightness agree.

On the same supposition, the value of B $(R^2 - 1)$ in 1898 would be 1·3 full moon, and the total brightness of the corona would appear as $1·1 + 1·3 = 2·4$ full moon.

Summary.

(*a.*) The brightness of the corona of 1898 at a point distant r from the sun's *centre* expressed in solar radii may be approximately represented by the formula

$$\text{brightness} = Ar^{-6} + B,$$

where A and B are constants.

(*b.*) The first term may be considered as corona proper, while B may be taken as representing the constant illumination of the sky, or glare. In 1898 the value of B was $2^{-6·4} = 0·012$ moon, taking the brightness of the moon as 0·02 candle at 1 foot.

(*c.*) The constant A varies with the radius along which measures are made. In 1898 it varied from $2^{0·0}$ moon to $2^{1·9}$ moon, the mean being $2^{·115}$ moon or 2·2 moon.

(*d.*) The same formula will fairly represent the 1893 corona, the mean value of A being $2^{0·23} = 1·2$, and the value of B $2^{-7·8} = 0·0046$.

(*e.*) The total brightness of the corona depends on the area of sky included. If a circular area 5° in diameter be included, the total brightness of the 1893 corona may be taken as 1·1 full moon, agreeing with the visual measures made, and that of 1898, on the same supposition, would be about 2·4 full moon.

"The Boiling Point of Liquid Hydrogen, determined by Hydrogen and Helium Gas Thermometers." By JAMES DEWAR, M.A., LL.D., F.R.S., Professor of Chemistry at the Royal Institution, and Jacksonian Professor, University of Cambridge. Received January 8,—Read February 7, 1901.

In a former paper† it was shown that a platinum-resistance thermometer gave for the boiling point of hydrogen $-238°·4$ C., or $34°·6$

* The dimensions of the box are not given, either here or in the previous paper to which we are referred; but on p. 369 of the 'Philosophical Transactions, A, 1889, there is a diagram of the box, from which it would appear that the angular aperture was not greater than 12°, judging by outside measurements.

† "On the Boiling Point of Liquid Hydrogen under Reduced Pressure," 'Roy. Soc. Proc.,' 1898 (vol. 64, p. 227).